

**Math 347: Homework 2**  
**Due on: Sep. 14, 2018**

1. (\*) Which of the following mathematical sentences are equivalent to each other:
  - (i)  $P \rightarrow Q$ ;
  - (ii)  $\neg(P \vee Q)$ ;
  - (iii)  $\neg(P \wedge Q)$ ;
  - (iv)  $P \wedge \neg Q$ ;
  - (v)  $\neg(P \rightarrow Q)$ ;
  - (vi)  $P \vee \neg Q$ ;
  - (vii)  $\neg P \vee \neg Q$ ;
  - (viii)  $\neg P \wedge \neg Q$ ;
  - (ix)  $\neg P \vee Q$ .
2. (\*) What can you say about the statement  $(P \wedge (P \rightarrow Q)) \rightarrow Q$ ? (If you are stuck, look up the definition of a truth table.)
3. For each statement below about the natural numbers, decide whether it is true or false and prove your claim or give a counter-example.
  - (i) If  $n \in \mathbb{N}$  and  $n^2 + (n + 1)^2 = (n + 2)^2$ , then  $n = 3$ .
  - (ii) For all  $n \in \mathbb{N}$ , it is false that  $(n - 1)^3 + n^3 = (n + 1)^3$ .
4. (\*) Let  $P(x)$  be the assertion " $x$  is odd", and  $Q(x)$  be the assertion " $x$  is twice an integer". Determine if the following are true or false:
  - (i)  $(\forall x \in \mathbb{Z})[P(x) \Rightarrow Q(x)]$ ;
  - (ii)  $(\forall x \in \mathbb{Z})[Q(x) \Rightarrow P(x)]$ ;
5. Let  $A, B$  and  $C$  be sets. Prove that  $(A \cup B) - C$  must be a subset of  $[A - (B \cup C)] \cup [B - (A \cap C)]$ , but that equality need not hold.
6. (\*) Let  $f$  be a function from  $\mathbb{R}$  to  $\mathbb{R}$ . Without using words of negation, write the meaning of " $f$  is not an increasing function".